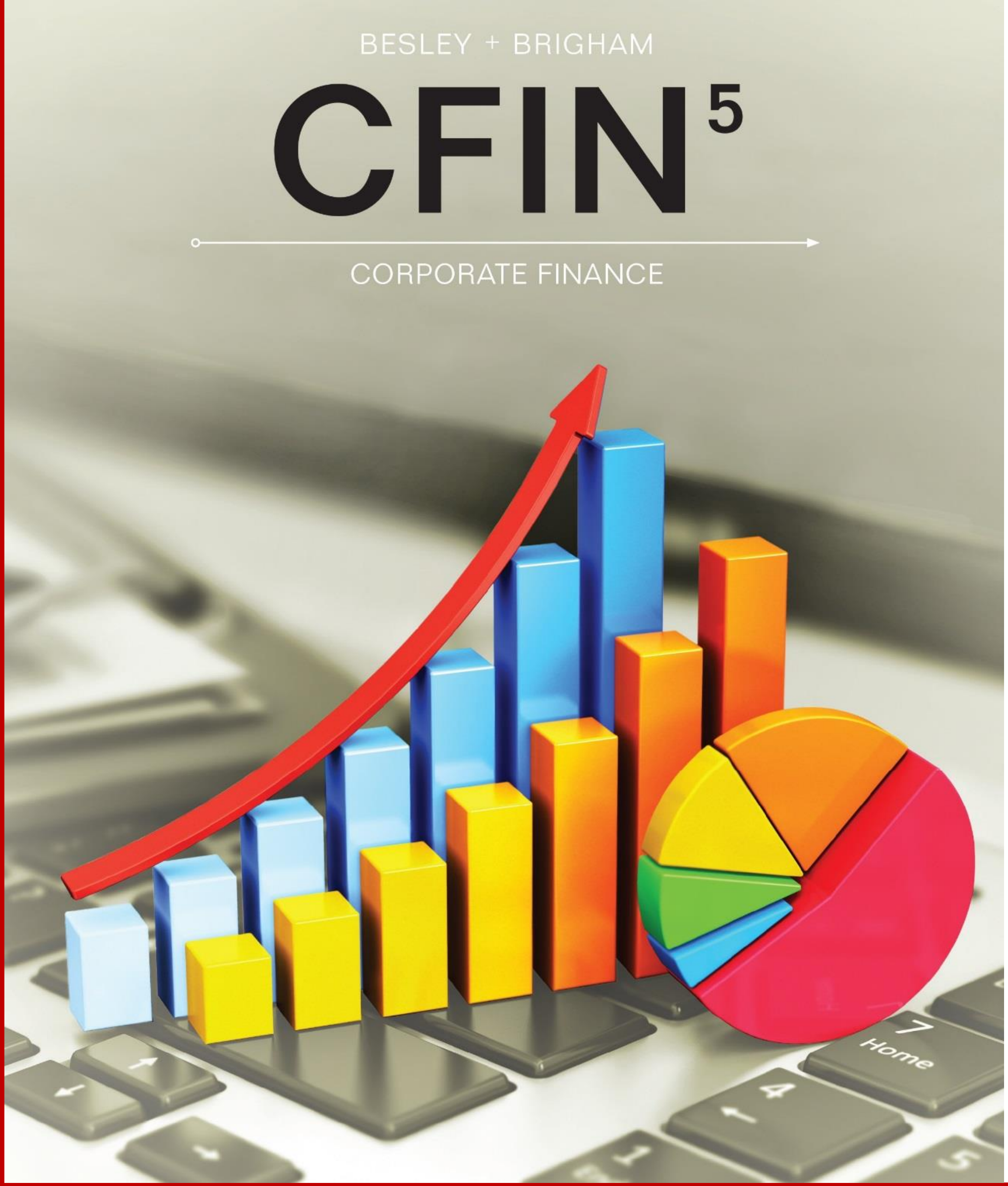


Chapter 4
Time Value of Money

Learning Outcomes

LO.1 Identify various types of cash flow patterns (streams) seen in business.

LO.2 Compute the future value of different cash flow streams. Explain the results.

LO.3 Compute the present value of different cash flow streams. Explain the results.

LO.4 Compute (a) the return (interest rate) on an investment (loan) and (b) how long it takes to reach a financial goal.

Learning Outcomes (cont.)

LO.5 Explain the difference between the Annual Percentage Rate (APR) and the Effective Annual Rate (EAR). Explain when it is appropriate to use each.

LO.6 Describe an amortized loan. Compute (a) amortized loan payments and (b) the amount that must be paid on an amortized loan at a specific date during the life of the loan.

Time Value of Money (TVM)

- The principles and computations used to revalue cash payoffs from different times so they are stated in dollars of the same time period.
- *Dollar amounts from different time periods should never be compared; rather, amounts should be compared only when they are stated in dollars at the same point in time, such as December 31 of a particular year.*
- Dollars from different time periods have opportunities to earn different amounts (numbers of periods) of interest (return).

Time Value of Money (TVM)

- At a 10 percent opportunity cost rate, which is better, receiving $700 today or receiving $935 in three years?

Time Value of Money (TVM)

- To answer the question, we must revalue the cash payoffs so they are stated in dollars at the same time period.

Year (n): 0 — 1 — 2 — 3

r = return = 10%

Cash flows:

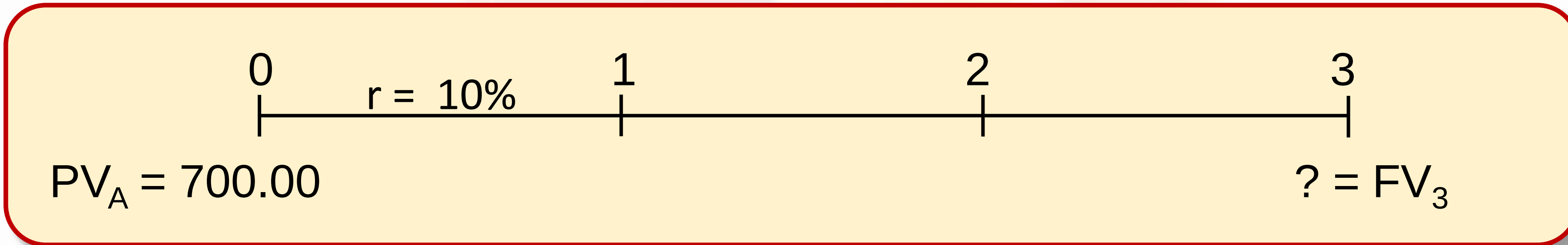
Option A: PV_A = \$700 → Translate the current \$700 into an FV amount by *adding* interest. → ? = FV_{A3}

OR

Option B: PV_B = ? ← Translate the future \$935 into a PV amount by *taking out* interest. ← \$935 = FV_{B3}

Cash Flow Time Lines

Graphical representations used to show timing of cash flows



PV = Present Value—the beginning amount that can be invested (current value of some future amount).

FV = Future Value—the value to which an amount invested today will grow at the end of n periods at an opportunity cost rate equal to r.

Types of Cash Flow Patterns

- Lump Sum Amount—a single payment (received or made) that occurs either today or at some date in the future.
- Annuity—multiple payments of the same amount over equal time periods.
- Uneven Cash Flows—multiple payments of different amounts over a period of time.

Future Value

- Compounding—to compute the future value of an amount we push forward the current amount by *adding* interest for each period in which the money can earn interest in the future.

$$
\begin{array}{ccccccc}
 & 0 & & 1 & & 2 & & 3 \\
 & \vert & \xrightarrow{r = 10\%} & \vert & \longrightarrow & \vert & \longrightarrow & \vert \\
PV_A = 700.00 & & \xrightarrow{\times 1.10} & 770.00 & \xrightarrow{\times 1.10} & 847.00 & \xrightarrow{\times 1.10} & 931.70 = FV_3
\end{array}
$$

Future Value of a Lump-Sum Amount FV_n

$$FV_n = PV(1 + r)^n$$

Four Ways to Solve Time Value of Money Problems

- Use a cash flow timeline
- Use an equation
- Use a financial calculator
- Use a spreadsheet

Free & Low Cost Excel Tutorial Options

- http://www.gcflearnfree.org/
- http://excelcentral.com/excel2013/basic/samplefilesmenu.aspx
- https://excelexposure.com/

FV_n Timeline Solution

- The Future Value of $700 invested at 10% per year for three (3) years

$$
\begin{array}{ccccccc}
0 & & 1 & & 2 & & 3 \\
 & r = 10\% & & & & & \\
PV_A = 700.00 & \xrightarrow{\times 1.10} & 770.00 & \xrightarrow{\times 1.10} & 847.00 & \xrightarrow{\times 1.10} & 931.70 = FV_3
\end{array}
$$

FV_n Equation Solution

$$FV_n = PV(1 + r)^n$$

$$= \$700(1.10)^3$$

$$= \$700(1.33100)$$

$$= \$931.70$$

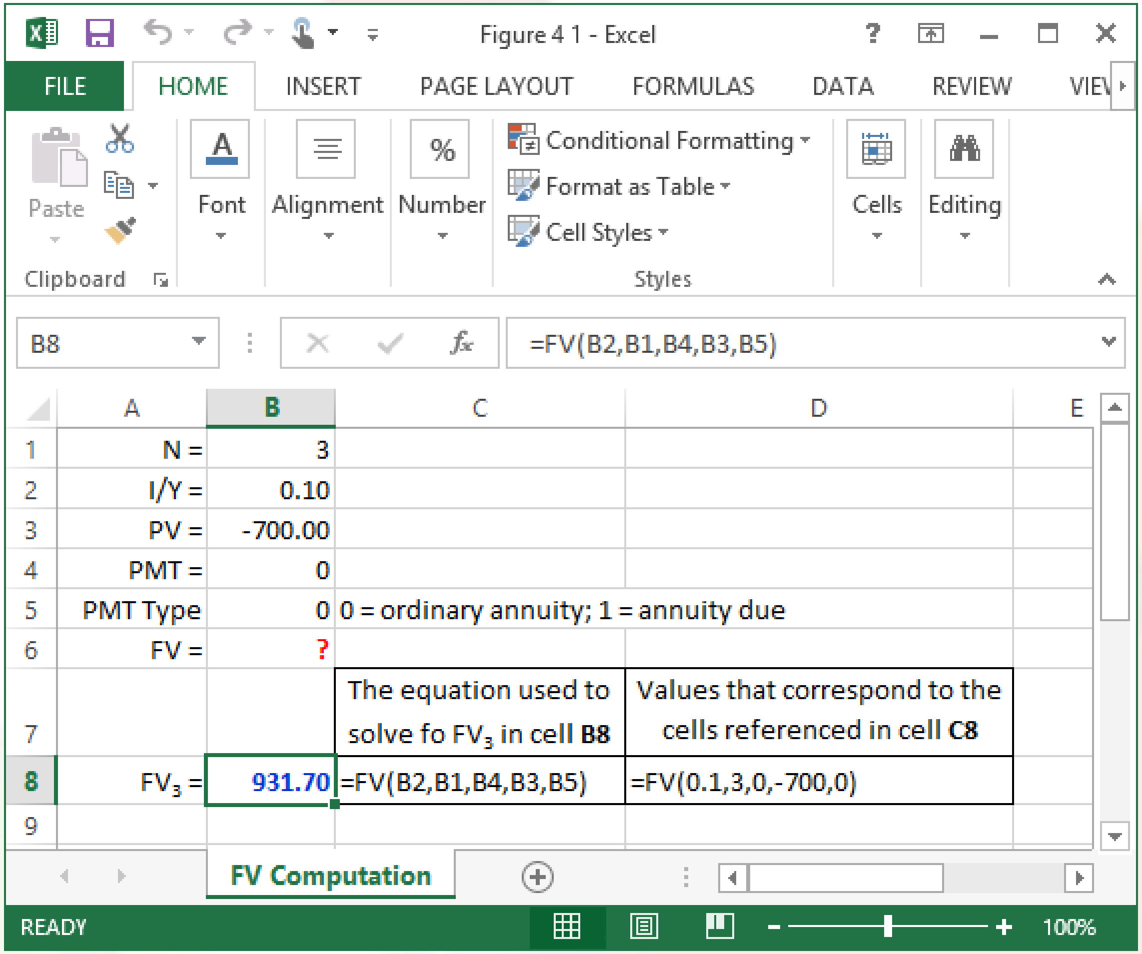
FV_n Financial Calculator Solution

Inputs:	3	10	-700	0	?
	N	I/Y	PV	PMT	FV
Output:					= 931.70

FV_n Spreadsheet Solution—MS Excel

- Set up a table that contains the data used to solve the problem.
- Click f_x and choose the FV function.
- Click the cells containing the appropriate data to enter the data into the FV function.
- Calculate the answer.

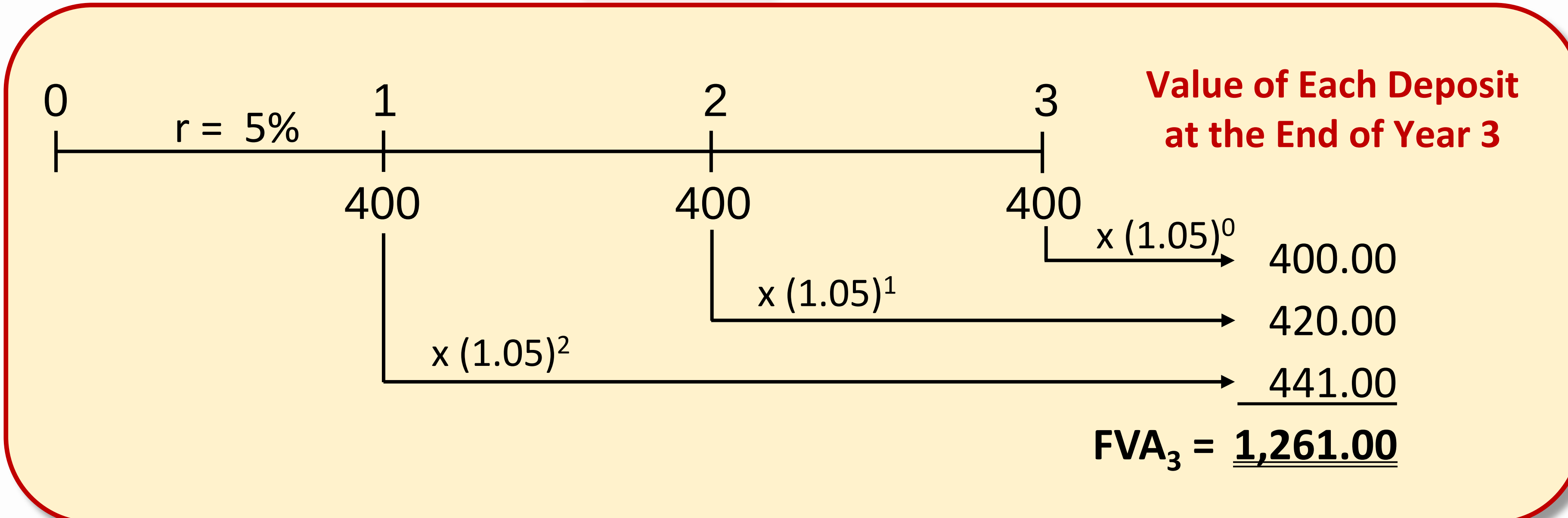
FV_n Spreadsheet Solution—MS Excel (cont.)



Future Value of an Annuity—FVA

- Annuity—a series of payments of equal amounts at fixed intervals for a specified number of periods.
  - Ordinary (deferred) Annuity—an annuity whose payments occur at the end of each period.
  - Annuity Due—an annuity whose payments occur at the beginning of each period.

What's the FV of a Three-year Ordinary Annuity of $400 at 5%?



FVA_n Equation Solution

$$FVA_n = PMT\left[(1+r)^{n-1} + (1+r)^{n-2} + \cdots + (1+r)^0\right]$$

$$= PMT\sum_{0}^{n-1}(1+r)^t$$

$$= PMT\left[\frac{(1+r)^n - 1}{r}\right]$$

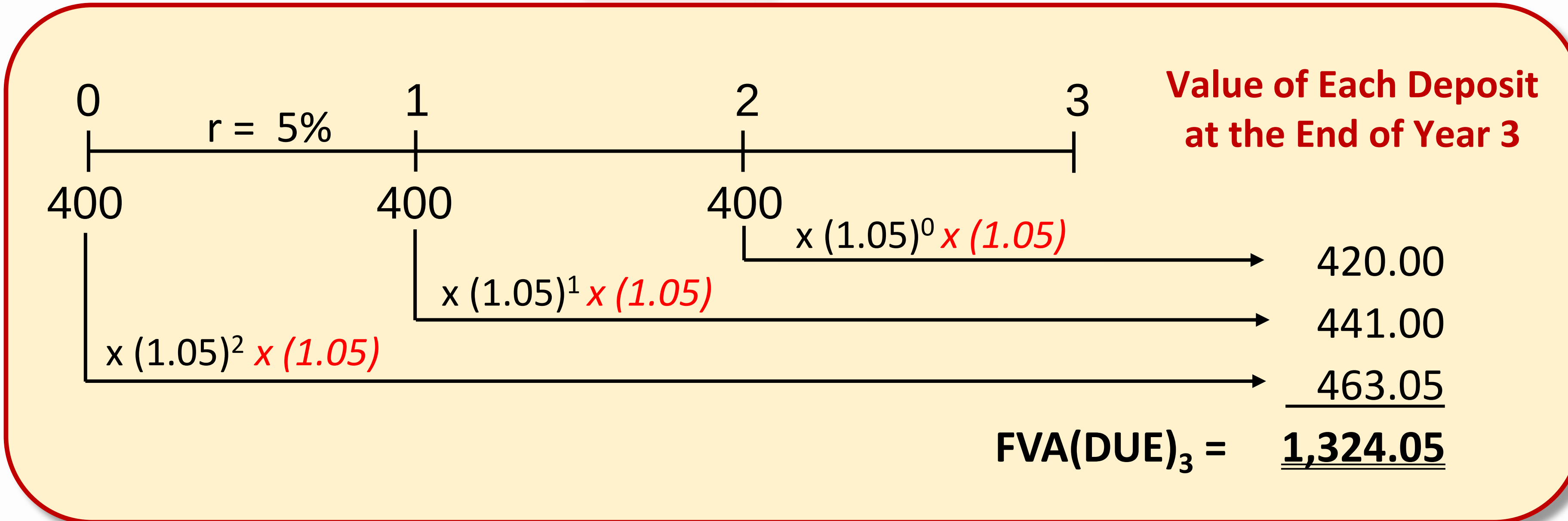
FVA_n Equation Solution (cont.)

$$FVA_3 = 400\left[\frac{(105)^3 - 1}{0.05}\right]$$

$$= 400(3.1525)$$

$$= 1,261.00$$

FV of an Annuity Due—FVA(DUE)$_n$



$FVA(DUE)_n$ Equation Solution

$$FVA(DUE)_n = PMT\left[(1+r)^n + (1+r)^{n-1} + \cdots + (1+r)^1\right]$$

$$= PMT\sum_{0}^{n-1}(1+r)^t \, (1+r)$$

$$= PMT\left\{\left[\frac{(1+r)^n - 1}{r}\right](1+r)\right\}$$

FVA(DUE)_n Equation Solution

$$\text{FVA(DUE)}_3 = 400\left\{\left[\frac{(105)^3 - 1}{0.05}\right](1.05)\right\}$$

$$= 400\{(3.1525)(1.05)\}$$

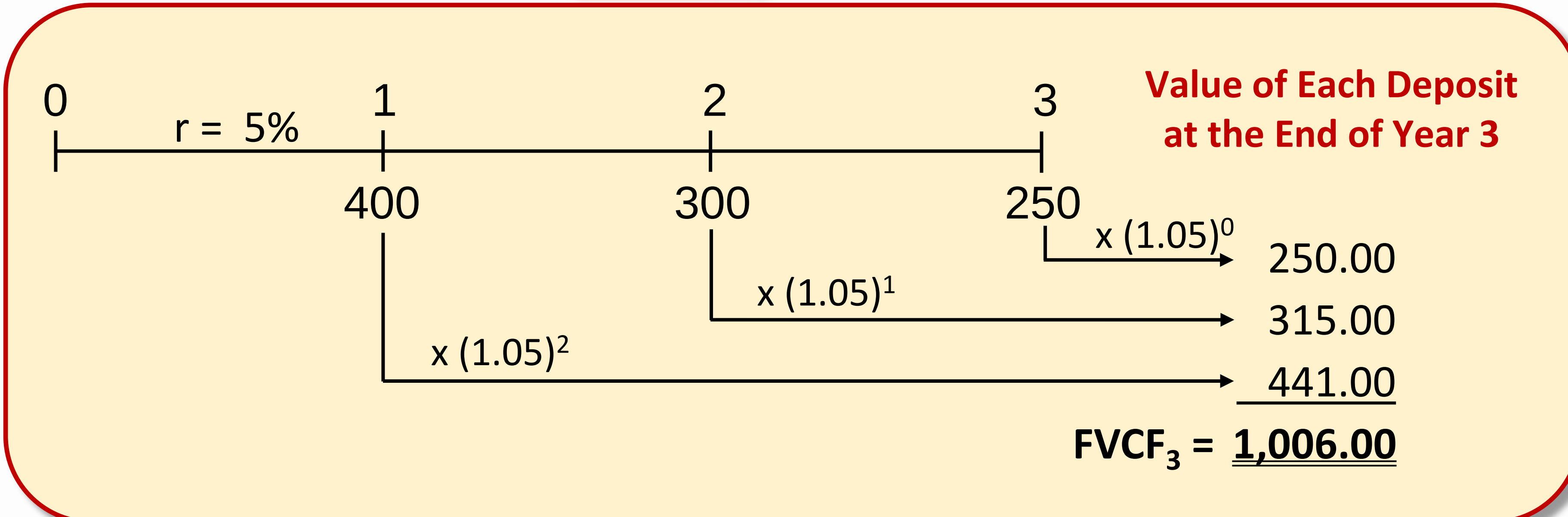
$$= 400(3.310125)$$

$$= 1{,}324.05$$

Cash Flow Streams

- Payment (PMT) designates constant cash flows—that is, an annuity stream.
- Cash flow (CF) designates cash flows in general, both constant cash flows (i.e., annuities) and uneven cash flows.

Find the FV of an Uneven Cash Flow Stream—$FVCF_n$



$FVCF_n$ Equation Solution

$$FVCF_n = CF_1(1+r)^{n-1} + \cdots + CF_n(1+r)^0$$

$$= \sum_{t=1}^{n} CF_t(1+r)^{n-t}$$

$FVCF_n$ Equation Solution

$$
\begin{aligned}
FVCF_3 &= 400(1.05)^2 + 300(1.05)^1 + 250(1.05)^0 \\
&= 400(1.1025) + 300(1.0500) + 250(1.0000) \\
&= 441.00 + 315.00 + 250.00 \\
&= 1{,}006.00
\end{aligned}
$$

Present Value

- Present value is the value today of a future cash flow or series of cash flows.
- *Discounting* is the process of finding the present value of a future cash flow or series of future cash flows
- Finding the present value (discounting) is the reverse of finding the future value (compounding).

Present Value—PV

- Discounting—to compute the present value of an amount we bring back to the present a future amount by *taking out* interest for each period in which the money can earn interest in the future.

0 — r = 10% — 1 — 2 — 3

$$PV_B = 702.48 \xleftarrow{\frac{1}{1.10}\ \times} 772.73 \xleftarrow{\frac{1}{1.10}\ \times} 850.00 \xleftarrow{\frac{1}{1.10}\ \times} 935.00 = FV_3$$

Present Value of a Lump-Sum Amount PV

$$PV = \frac{FV_n}{(1+r)^n} = FV_n\left[\frac{1}{(1+r)^n}\right]$$

Present Value of a Lump-Sum Amount PV

What is the PV of $935 due in Three (3) years if r = 10%?

$$PV = 935\left[\frac{1}{(1.10)^3}\right]$$

$$= 935(0.751314801)$$

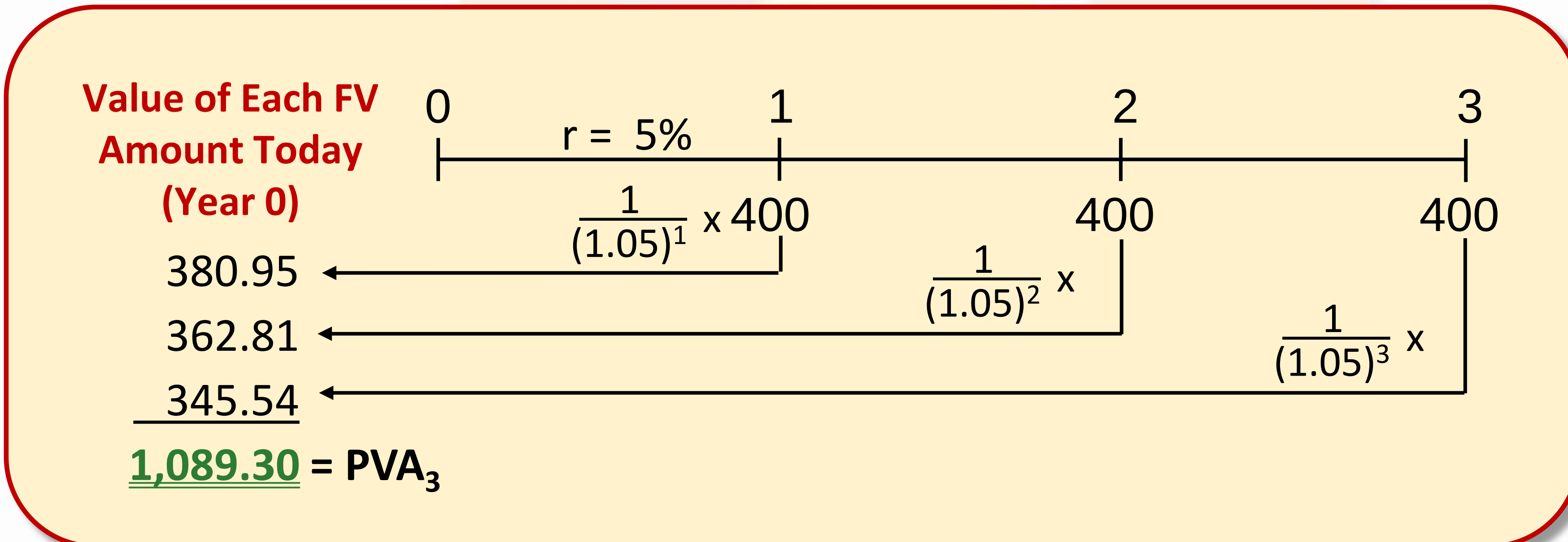
$$= 702.48$$

Present Value of an Annuity (Ordinary)—PVA_n

- PVA_n = the present value of an annuity with n payments.
- Each payment is discounted, and the sum of the discounted payments is the present value of the annuity.

PVA_n Cash Flow Solution

What is the PV of a three-year $400 ordinary annuity if r = 5%?



PVA_n Equation Solution

$$PVA_n = PMT\left[\frac{1}{(1+r)^1}\right] + \ldots + PMT\left[\frac{1}{(1+r)^n}\right]$$

$$= PMT\left[\sum_{t=1}^{n}\frac{1}{(1+r)^t}\right]$$

$$= PMT\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]$$

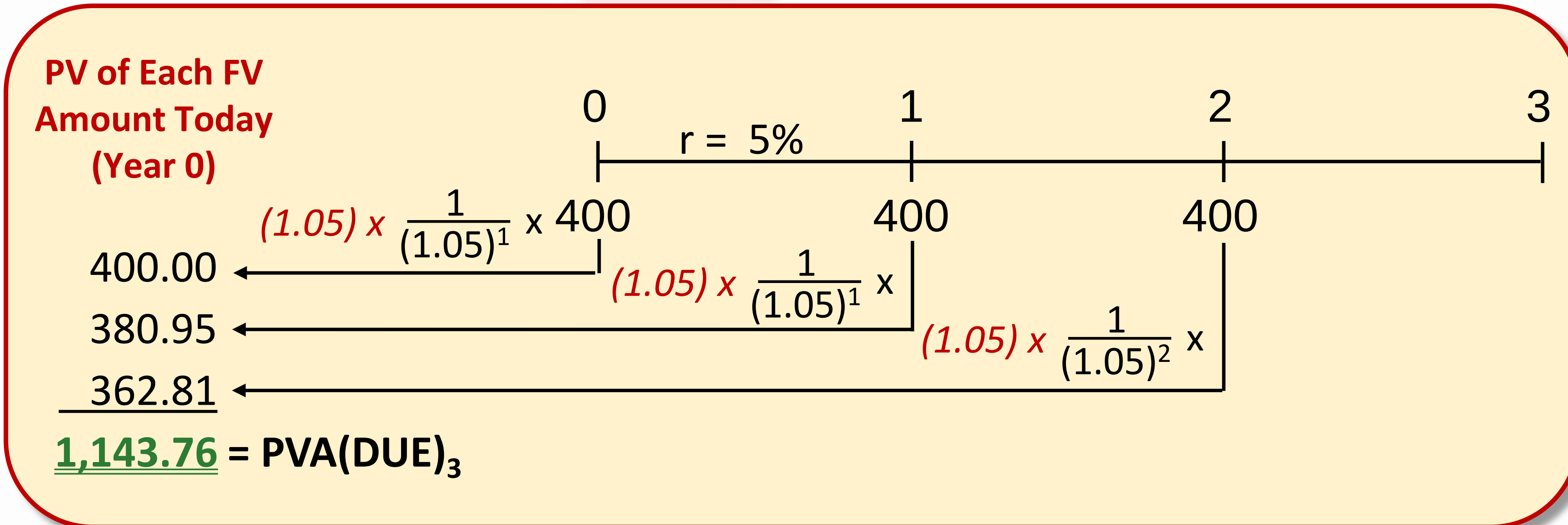
PVA_n Equation Solution

$$PVA_3 = 400\left[\frac{1 - \frac{1}{(1.05)^3}}{0.05}\right]$$

$$= 400(2.723248)$$

$$= 1{,}089.30$$

Present Value of an Annuity Due—PVA(DUE)$_n$



$PVA(DUE)_n$ Equation Solution

$$PVA(DUE)_n = PMT\left[\frac{1}{(1+r)^0}\right] + \ldots + PMT\left[\frac{1}{(1+r)^{n-1}}\right]$$

$$= PMT\left[\sum_{t=1}^{n}\frac{1}{(1+r)^t}\right](1+r)$$

$$= PMT\left\{\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right](1+r)\right\}$$

$PVA(DUE)_n$ Equation Solution

$$PVA(DUE)_3 = 400\left\{\left[\frac{1-\frac{1}{(1.05)^3}}{0.05}\right](1.05)\right\}$$

$$= 400\{(2.723248)(1.05)\}$$

$$= 1{,}143.76$$

Perpetuities—PVP

- Streams of equal payments that are expected to go on forever

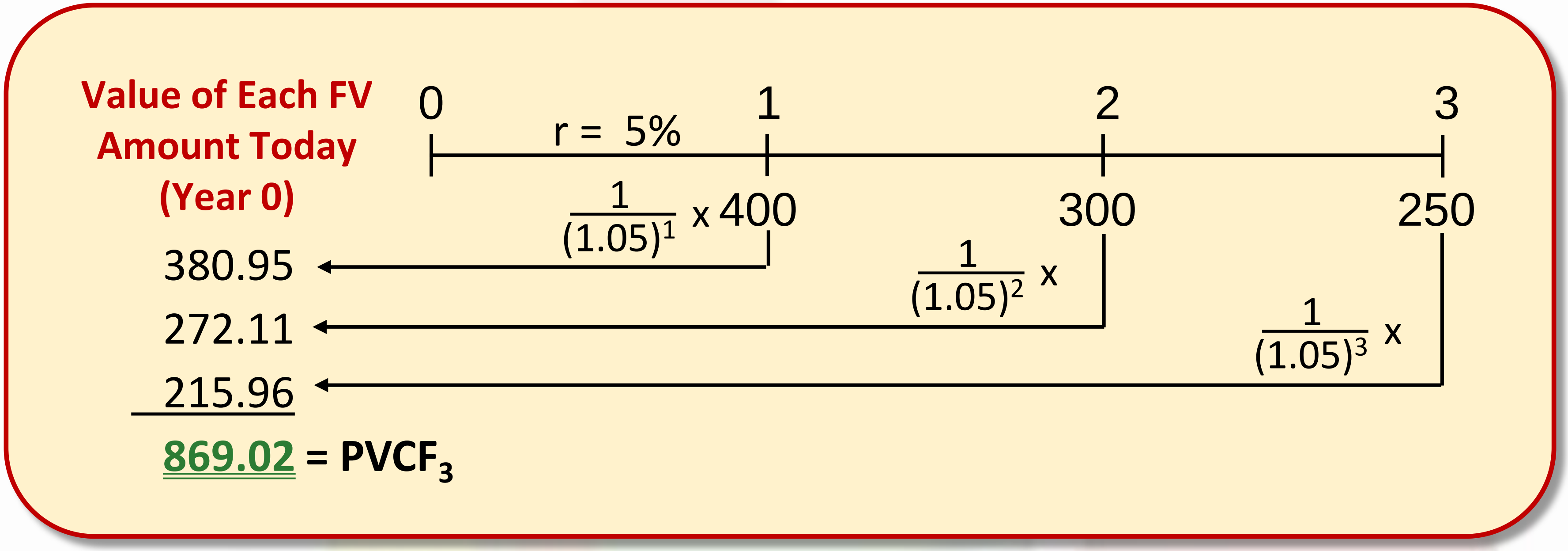
$$PVP = PMT\left[\frac{1}{r}\right] = \frac{PMT}{r}$$

Perpetuities—PVP

$$PVP_1 = \frac{100}{0.05} = 2{,}000$$

$$PVP_2 = \frac{100}{0.10} = 1{,}000$$

PV of an Uneven Cash Flow Stream—$PVCF_n$



$PVCF_n$ Equation Solution

$$PVCF_n = CF_1\left[\frac{1}{(1+r)^1}\right] + \cdots + CF_n\left[\frac{1}{(1+r)^n}\right]$$

$$= \sum_{t=1}^{n} CF_t\left[\frac{1}{(1+r)^t}\right]$$

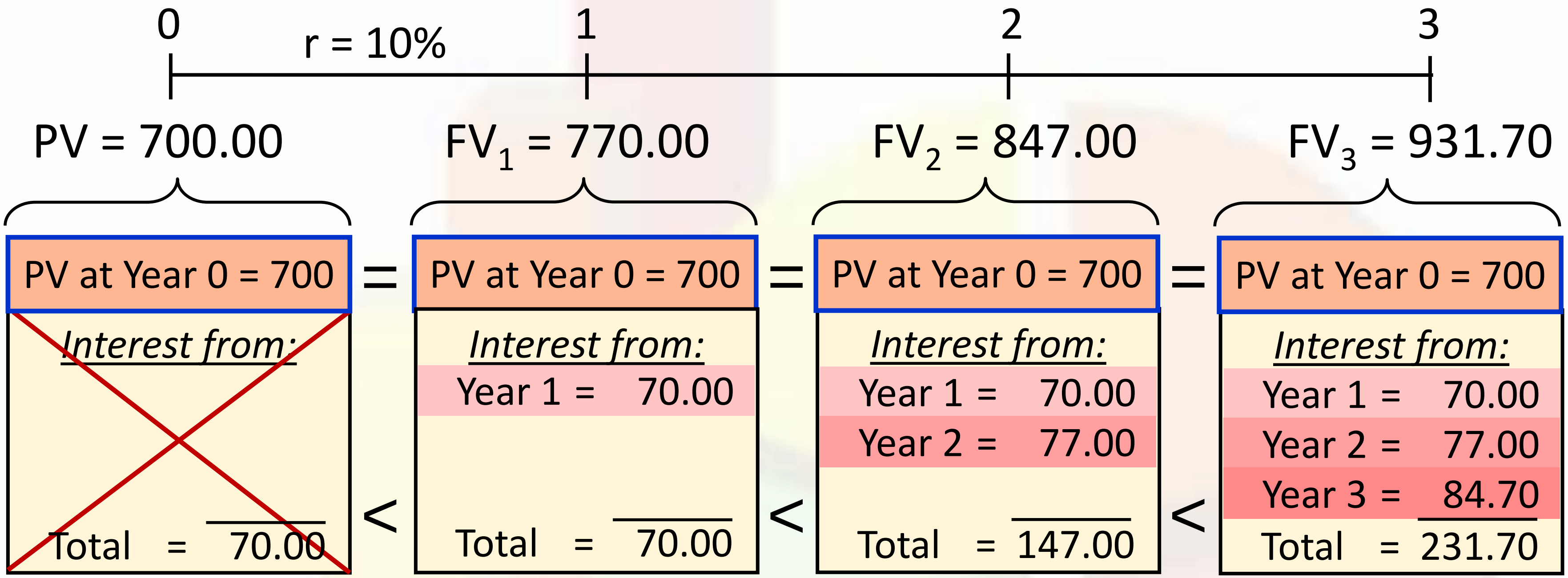
$PVCF_n$ Equation Solution

$$
\begin{aligned}
PVCF_3 &= 400\left[\frac{1}{(1.05)^1}\right] + 300\left[\frac{1}{(1.05)^2}\right] + 250\left[\frac{1}{(1.05)^3}\right] \\
&= 400(0.952381) + 300(0.907029) + 250(0.863838) \\
&= 380.95 + 272.11 + 215.96 \\
&= 869.02
\end{aligned}
$$

Comparison of FV with PV

- FV contains interest, whereas PV does not.
- At an opportunity cost rate of 10 percent:
  - a lump-sum payment of $700 *today* is the same as a lump-sum payment of $931.70 *in three years*.
  - The PV of $700 has no interest; the FV of $931.70 contains three years of interest, which equals $231.70.

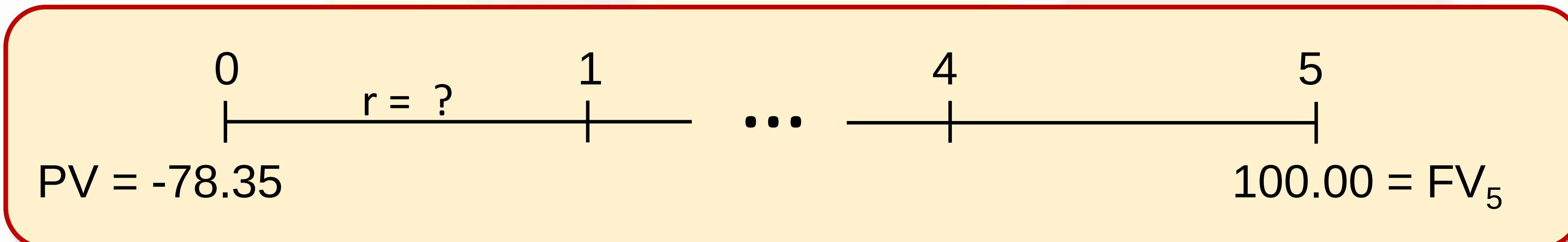
Comparison of FV with PV (cont.)



- The values given under the tick marks for each year differ only because they contain different amounts of interest.

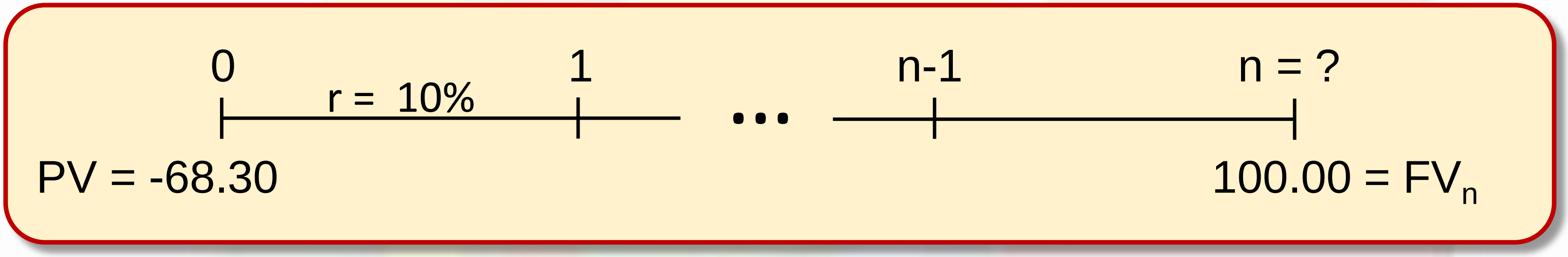
Solving for Interest Rates (r)

- Suppose you pay $78.35 for an investment that promises to pay you $100 five years from today. What annual rate of return will you earn on your investment?



Solving for Time (n)

- A security that costs $68.30 will provide a return of 10 percent per year. If you want to keep the investment until it grows to a value of $100, how long will you have to keep it?



Semiannual and Other Compounding Periods

- Annual compounding is the process of determining the future value of a cash flow or series of cash flows when interest is earned (added) once per year.
- Semiannual compounding is the process of determining the future value of a cash flow or series of cash flows when interest is added twice per year.

FV of a lump sum

- The FV of a lump sum be larger if interest is compounded more often, holding the stated r constant? Why?
  - If compounding is more frequent than once per year—for example, semi-annually, quarterly, or daily—interest is earned on interest—that is, compounded—more often.
  - Compared to annual compounding, a greater amount of interest is earned when interest is compounded more than once per year.

Distinguishing Between Different Interest Rates

r_{SIMPLE} = Simple (Quoted) Rate
used to compute the interest paid per period

APR = Annual Percentage Rate = r_{SIMPLE}

r_{EAR} = Effective Annual Rate
the annual rate of interest actually being earned

Comparison of Different Types of Interest Rates

- r_{SIMPLE}: Written into contracts, quoted by banks and brokers. Not used in calculations or shown on time lines.
- r_{PER}: Interest rate per period (e.g., per year, per month, etc.); used in calculations; shown on time lines.
- r_{EAR}: Used to compare returns on investments with different payments per year.

Simple (Quoted) Rate

- r_{SIMPLE} is stated in contracts
  Periods per year (m) must also be given

- Examples:
  - 8%, compounded quarterly
  - 8%, compounded daily (365 days)

Periodic Rate, r_{PER}

- Periodic rate = r_{PER} = r_{SIMPLE}/m, where m is number of compounding periods per year; m = 4 for quarterly compounding, 12 for monthly compounding, and so forth.
- Examples:
  - 8%, compounded quarterly: r_{PER} = 8%/4 = 2%
  - 8%, compounded monthly: r_{PER} = 8%/12 = 0.667%

Effective Annual Rate

- The annual rate that causes PV to grow to the same FV as it would with multi-period compounding.

$$\text{Effective Annual Rate (EAR)} = r_{EAR} = \left(1 + \frac{r_{SIMPLE}}{m}\right)^m - 1.0$$

$$= (1 + r_{PER})^m - 1.0$$

Computing r_{EAR}

- What is the effective annual return (EAR) for an investment that pays 12 percent interest, compounded monthly?

$$r_{EAR} = \left(1 + \frac{r_{SIMPLE}}{m}\right)^m - 1.0$$

$$= \left(1 + \frac{0.12}{12}\right)^{12} - 1.0 = (1.01)^{12} - 1.0$$

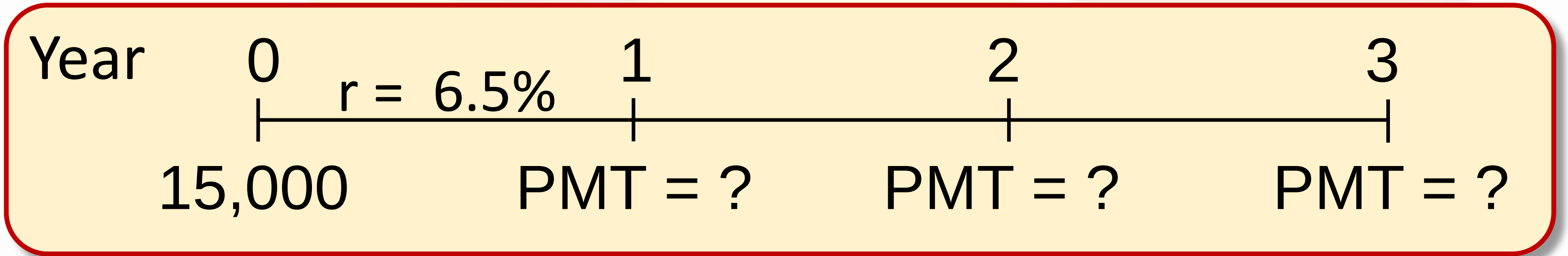
$$= 0.1268 = 12.68\%$$

Amortized Loans

- Amortized Loan—a loan that is repaid in equal payments over its life.
- A portion of the payment represents interest and the remainder represents repayment of the amount that was borrowed.
- Amortization schedules are widely used for home mortgages, auto loans, and so forth to determine how much of each payment represents principal repayment and how much represents interest.

Amortization schedule

- An amortization schedule for a $33,000, 6.5 percent loan that requires three equal annual payments.



Amortization Schedule

Year	Beginning of Year Balance (1)	Payment (2)	Interest @ 6.5% (3) = (1) x 0.065	Repayment of Principal (4) = (2) – (3)	Remaining Balance (5) = (1) – (4)
1	$33,000.00	$12,460	$2,145.00	$10,315.00	$22,685.00
2	22,685.00	12,460	1,474.53	10,985.48	11,699.53
3	11,699.53	12,460	760.47	11,699.53	0.00